FSE of Continuous Time Signals:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(\frac{2\pi}{T})t}$$
$$a_k = \frac{1}{T}\int_T x(t)e^{-jkw_0 t}dt = \frac{1}{T}\int_T x(t)e^{-jk(\frac{2\pi}{T})t}dt \quad (1)$$

FSE of Discrete Time Signals:

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jkw_0 n} = \sum_{k=\langle N\rangle} a_k e^{jk(\frac{2\pi}{N})n}$$
$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jkw_0 n} = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk(\frac{2\pi}{N})n}dt$$

Discrete-Time Fourier Transform:

$$x[n] = \frac{1}{2\pi}\int_{<2\pi>} X(j\Omega)e^{j\Omega n}d\Omega$$
$$X(j\Omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\Omega n}$$

Continuous-Time Fourier Transform:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(jw)e^{jwt}dw$$
$$X(jw) = \int_{-\infty}^{+\infty} x(t)e^{-jwt}dt$$

Trigonometric Identities/Definitions:

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan(u \pm \tan v)}{1 \mp \tan u \tan v}$$
$$\sin u + \sin v = 2\sin(\frac{u+v}{2})\cos(\frac{u-v}{2})$$
$$\sin u - \sin v = 2\cos(\frac{u+v}{2})\sin(\frac{u-v}{2})$$
$$\cos u + \cos v = 2\cos(\frac{u+v}{2})\cos(\frac{u-v}{2})$$
$$\cos u - \cos v = -2\sin(\frac{u+v}{2})\sin(\frac{u-v}{2})$$
$$sinc(x) = \frac{\sin \pi x}{\pi x}$$

THESE TABLES ARE NOT MEANT TO BE COMPLETE BUT RATHER A SUPPORTING MATERIAL.

Time Domain	**CTFT Domain**
$\sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(w - kw_0)$
$e^{jw_0 t}$	$2\pi\delta(w - w_0)$
$\cos w_0 t$	$\pi[\delta(w - w_0) + \delta(w + w_0)]$
$\sin w_0 t$	$\frac{\pi}{j}[\delta(w - w_0) - \delta(w + w_0)]$
$x(t) = 1$	$2\pi\delta(w)$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta(w - \frac{2\pi k}{T})$
$rect_{T_1}(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$2T_1 sinc(\frac{wT_1}{\pi}) = \frac{2\sin wT_1}{w}$
$\frac{W}{\pi} sinc(\frac{Wt}{\pi}) = \frac{\sin Wt}{\pi t}$	$X(jw) = \begin{cases} 1, & \lvert w\rvert < W \\ 0, & \lvert w\rvert > W \end{cases}$
$\delta(t)$	1
$u(t)$	$\frac{1}{jw} + \pi\delta(w)$
$\delta(t - t_0)$	e^{-jwt_0}
$e^{-at}u(t), \Re e\{a\} > 0$	$\frac{1}{a+jw}$
$te^{-at}u(t), \Re e\{a\} > 0$	$\frac{1}{(a+jw)^2}$
$\frac{t^{n-1}}{(n-1)!} e^{-at}u(t), \Re e\{a\} > 0$	$\frac{1}{(a+jw)^n}$

Aperiodic signal	**Fourier Transform**
$ax(t) + by(t)$	$aX(jw) + bY(jw))$
$x(t - t_0)$	$e^{-jwt_0} X(jw)$
$e^{jw_0 t} x(t)$	$X(j(w - w_0))$
$x^*(t)$	$X^*(-jw)$
$x(-t)$	$X(-jw)$
$x(at)$	$\frac{1}{\lvert a\rvert} X(\frac{jw}{a})$
$x(t) * y(t)$	$X(jw)Y(jw)$
$x(t)y(t)$	$\frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta)Y(j(w - \theta))d\theta$
$\frac{d}{dt}x(t)$	$jwX(jw)$
$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{jw} X(jw) + \pi X(0)\delta(w)$
$tx(t)$	$j\frac{d}{dw} X(jw)$
$x(t) \in \Re$	$\begin{cases} X(jw) = X^*(-jw) \\ \Re e\{X(jw)\} = \Re e\{X(-jw)\} \\ \Im m\{X(jw)\} = -\Im m\{X(-jw)\} \\ \mid X(jw) \mid = \mid X(-jw) \mid \\ \sphericalangle X(jw) = -\sphericalangle X(-jw) \end{cases}$

Time Domain	DTFT Domain
$\sum_{k=\langle N\rangle} a_k e^{jk(\frac{2\pi}{N})n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\Omega - \frac{2\pi k}{N})$
$e^{j\Omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\Omega - \Omega_0 - 2\pi l)$
$\cos \Omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\Omega - \Omega_0 - 2\pi l) + \delta(\Omega + \Omega_0 - 2\pi l)\}$
$\sin \Omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\Omega - \Omega_0 - 2\pi l) - \delta(\Omega + \Omega_0 - 2\pi l)\}$
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\Omega - 2\pi l)$
Periodic square wave $x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & N_1 < \lvert n\rvert \le \frac{N}{2} \end{cases}$ $x[n+N] = x[n]$	$\sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\Omega - \frac{2\pi k}{N})$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta(\Omega - \frac{2\pi k}{N})$
$a^n u[n], \ \lvert a\rvert < 1$	$\frac{1}{1 - ae^{-j\Omega}}$
$x[n] = \begin{cases} 1, & \lvert n\rvert \le N_1 \\ 0, & \lvert n\rvert > N_1 \end{cases}$	$\frac{\sin[\Omega(N_1 + \frac{1}{2})]}{\sin \Omega/2}$
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} sinc(\frac{Wn}{\pi})$ $0 < W < \pi$	$X(\Omega) = \begin{cases} 1, & 0 \le \lvert\Omega\rvert \le W \\ 0, & W < \lvert\Omega\rvert \le \pi \end{cases}$
$\delta[n]$	1
$u[n]$	$\frac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{+\infty} \pi \delta(\Omega - 2\pi k)$
$\delta(n - n_0)$	$e^{-j\Omega n_0}$
$(n+1)a^n u[n], \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\Omega})^2}$
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], \lvert a\rvert < 1$	$\frac{1}{(1 - ae^{-j\Omega})^r}$

Aperiodic signal	DTFT
$ax[n] + by[n]$	$aX(e^{j\Omega}) + bY(e^{j\Omega}))$
$x[n - n_0]$	$e^{-j\Omega n_0} X(e^{j\Omega})$
$e^{j\Omega_0 n} x[n]$	$X(e^{j(\Omega - \Omega_0)})$
$x^*[n]$	$X^*(e^{-j\Omega})$
$x[-n]$	$X(e^{-j\Omega})$
$x_k[n] = \begin{cases} x[n/k], & \text{if n=multiple of k} \\ 0, & \text{if } n \ne \text{ multiple of k} \end{cases}$	$X(e^{jk\Omega})$
$x[n] * y[n]$	$X(e^{j\Omega})Y(e^{j\Omega})$
$x[n]y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(e^{j\theta}) Y(e^{j(\Omega - \theta)}) d\theta$
$x[n] - x[n-1]$	$(1 - e^{j\Omega})X(e^{j\Omega})$
$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - e^{-j\Omega}} X(e^{j\Omega})$
$nx[n]$	$j \frac{d}{d\Omega} X(e^{j\Omega})$
$x[n] \in \Re$	$\begin{cases} X(e^{j\Omega}) = X^*(e^{-j\Omega}) \\ \Re e\{X(e^{j\Omega})\} = \Re e\{X(e^{-j\Omega})\} \\ \Im m\{X(e^{j\Omega})\} = -\Im m\{X(e^{-j\Omega})\} \\ \mid X(e^{j\Omega}) \mid = \mid X(e^{-j\Omega}) \mid \\ \sphericalangle X(e^{j\Omega}) = -\sphericalangle X(e^{-j\Omega}) \end{cases}$

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TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$x(t)$, $y(t)$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k, b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t)$, $\alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$a_k = a^*_{-k}$ $\mathcal{Re}\{a_k\} = \mathcal{Re}\{a_{-k}\}$ $\mathcal{Im}\{a_k\} = -\mathcal{Im}\{a_{-k}\}$ $\|a_k\| = \|a_{-k}\|$ $\sphericalangle a_k = -\sphericalangle a_{-k}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$x_e(t) = \mathcal{Ev}\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{Od}\{x(t)\}$ [$x(t)$ real]	$\mathcal{Re}\{a_k\}$ $j\mathcal{Im}\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix} a_k \\ b_k \end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k \begin{pmatrix} \text{viewed as periodic} \\ \text{with period } mN \end{pmatrix}$
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k] \begin{pmatrix} \text{finite valued and periodic only} \\ \text{if } a_0 = 0 \end{pmatrix}$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right) a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N} \sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$

Some Laplace Transforms		
Signal	**Transform**	**ROC**
$\delta(t)$	1	**All s**
$u(t)$	$\frac{1}{s}$	$\Re\{s\} > 0$
$-u(-t)$	$\frac{1}{s}$	$\Re\{s\} < 0$
$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re\{s\} > 0$
$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re\{s\} < 0$
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\Re\{s\} > -a$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\Re\{s\} < -a$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(s+a)^n}$	$\Re\{s\} > -a$
$-\frac{t^{n-1}}{(n-1)!}e^{-at}u(-t)$	$\frac{1}{(s+a)^n}$	$\Re\{s\} < -a$
$\delta(t-T)$	e^{-Ts}	**All s**
$[\cos w_0 t]u(t)$	$\frac{s}{s^2+w_0^2}$	$\Re\{s\} > 0$
$[\sin w_0 t]u(t)$	$\frac{w_0}{s^2+w_0^2}$	$\Re\{s\} > 0$
$[e^{-at}\cos w_0 t]u(t)$	$\frac{s+a}{(s+a)^2+w_0^2}$	$\Re\{s\} > -a$
$[e^{-at}\sin w_0 t]u(t)$	$\frac{w_0}{(s+a)^2+w_0^2}$	$\Re\{s\} > -a$
$u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	**All s**
$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{\textbf{n times}}$	$\frac{1}{s^n}$	$\Re\{s\} > 0$

Properties of The Laplace Transform		
Signal	**Laplace Transform**	**ROC**
$ax_1 + bx_2$	$aX_1(s) + bX_2(s)$	**At least** $R_1 \bigcap R_2$
$x(t-t_0)$	$e^{-st_0}X(s)$	**R**
$e^{s_0 t}x(t)$	$X(s-s_0)$	**Shifted version of R**
$x(at)$	$\frac{1}{\lvert a \rvert}X(\frac{s}{a})$	**Scaled ROC**
$x^*(t)$	$X^*(s^*)$	**R**
$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	**At least** $R_1 \bigcap R_2$
$\frac{d}{dt}x(t)$	$sX(s)$	**At least R**
$-tx(t)$	$\frac{d}{ds}X(s)$	**R**
$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	**At least** $R \bigcap \{Re\{s\} > 0\}$

TABLE 10.1 PROPERTIES OF THE z-TRANSFORM

Section	Property	Signal	z-Transform	ROC
		$x[n]$	$X(z)$	R
		$x_1[n]$	$X_1(z)$	R_1
		$x_2[n]$	$X_2(z)$	R_2
10.5.1	Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of R_1 and R_2
10.5.2	Time shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	R, except for the possible addition or deletion of the origin
10.5.3	Scaling in the z-domain	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	R
		$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$z_0 R$
		$a^n x[n]$	$X(a^{-1}z)$	Scaled version of R (i.e., $\|a\|R$ = the set of points $\{\|a\|z\}$ for z in R)
10.5.4	Time reversal	$x[-n]$	$X(z^{-1})$	Inverted R (i.e., R^{-1} = the set of points z^{-1}, where z is in R)
10.5.5	Time expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ for some integer r	$X(z^k)$	$R^{1/k}$ (i.e., the set of points $z^{1/k}$, where z is in R)
10.5.6	Conjugation	$x^*[n]$	$X^*(z^*)$	R
10.5.7	Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of R_1 and R_2
10.5.7	First difference	$x[n] - x[n-1]$	$(1 - z^{-1})X(z)$	At least the intersection of R and $\|z\| > 0$
10.5.7	Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - z^{-1}}X(z)$	At least the intersection of R and $\|z\| > 1$
10.5.8	Differentiation in the z-domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R
10.5.9		Initial Value Theorem If $x[n] = 0$ for $n < 0$, then $x[0] = \lim_{z\to\infty} X(z)$		

TABLE 10.2 SOME COMMON z-TRANSFORM PAIRS

Signal	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
3. $-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
4. $\delta[n-m]$	z^{-m}	All z, except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$\|z\| > \|\alpha\|$
6. $-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\|z\| < \|\alpha\|$
7. $n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\|z\| > \|\alpha\|$
8. $-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\|z\| < \|\alpha\|$
9. $[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\|z\| > 1$
10. $[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\|z\| > 1$
11. $[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\|z\| > r$
12. $[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\|z\| > r$

TABLE 10.3 PROPERTIES OF THE UNILATERAL z-TRANSFORM

Property	Signal	Unilateral z-Transform
—	$x[n]$	$\mathcal{X}(z)$
—	$x_1[n]$	$\mathcal{X}_1(z)$
—	$x_2[n]$	$\mathcal{X}_2(z)$
Linearity	$ax_1[n]+bx_2[n]$	$a\mathcal{X}_1(z)+b\mathcal{X}_2(z)$
Time delay	$x[n-1]$	$z^{-1}\mathcal{X}(z)+x[-1]$
Time advance	$x[n+1]$	$z\mathcal{X}(z)-zx[0]$
Scaling in the z-domain	$e^{j\omega_0 n}x[n]$ $z_0^n x[n]$ $a^n x[n]$	$\mathcal{X}(e^{-j\omega_0}z)$ $\mathcal{X}(z/z_0)$ $\mathcal{X}(a^{-1}z)$
Time expansion	$x_k[n] = \begin{cases} x[m], & n = mk \\ 0, & n \neq mk \end{cases}$ for any m	$\mathcal{X}(z^k)$
Conjugation	$x^*[n]$	$\mathcal{X}^*(z^*)$
Convolution (assuming that $x_1[n]$ and $x_2[n]$ are identically zero for $n < 0$)	$x_1[n] * x_2[n]$	$\mathcal{X}_1(z)\mathcal{X}_2(z)$
First difference	$x[n]-x[n-1]$	$(1-z^{-1})\mathcal{X}(z)-x[-1]$
Accumulation	$\sum_{k=0}^{n} x[k]$	$\frac{1}{1-z^{-1}}\mathcal{X}(z)$
Differentiation in the z-domain	$nx[n]$	$-z\frac{d\mathcal{X}(z)}{dz}$

Initial Value Theorem

$$x[0] = \lim_{z\to\infty}\mathcal{X}(z)$$